Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 1

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

A salesperson with an annual salary of $\$89\,400$ is paid monthly.

(a) Calculate their monthly pay, before any deductions are made. (1 mark)

Solution
$89\,400 \div 12 = \$7450$
Specific behaviours
✓ correct wage

(b) Every month their employer withholds 27% of the amount that their monthly pay exceeds $\$2000$ for tax. Determine the monthly tax withheld. (2 marks)

Solution
$7450 - 2000 = 5450$ Tax: $5450 \times 0.27 = \$1471.50$
Specific behaviours
✓ difference ✓ calculates percentage

(c) Their employer contributes an amount equal to 9.5% of their annual salary into a superannuation fund each year. Calculate this annual contribution. (1 mark)

Solution
Contribution: $89\,400 \times 0.095 = \$8493$
Specific behaviours
✓ calculates percentage

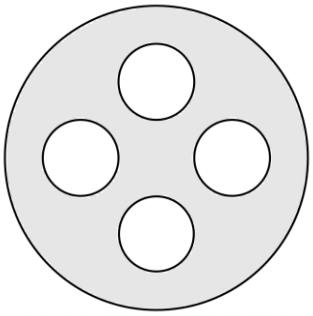
(d) The salesperson is offered an alternative contract where they are paid a reduced salary of $\$36\,000$ plus a commission of 1.75% on all sales they make. If the salesperson expects to make sales worth $\$2\,750\,000$ over the next year, determine whether they will earn more or less if they switch to the alternative contract. Justify you answer. (2 marks)

Solution
$36\,000 + 2\,750\,000 \times 0.0175 = 36\,000 + 48\,125$ $= \$84\,125$ Salesperson will earn less with new contract.
Specific behaviours
✓ calculates commission ✓ calculates yearly earnings and states less

See next page

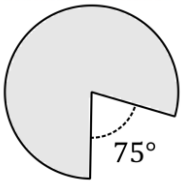
Question 10 **(8 marks)**

(a) A design for a button consists of a circle of radius 17 mm with four circles of radius 5 mm removed, as shown below. Determine the area of the shaded region. (3 marks)



Solution
$A_L = \pi(17)^2 = 908$ $A_S = \pi(5)^2 = 78.5$ $4A_S = 314$ Shaded area: $A = 908 - 314 = 594 \text{ mm}^2$
Specific behaviours
✓ area of large circle ✓ area of 4 small circles ✓ calculates shaded area

(b) Another design consists of a sector of a circle of diameter 16 mm, as shown below.



(i) Determine the area of the sector. (3 marks)

Solution
$\theta = 360 - 75 = 285, \quad r = 16 \div 2 = 8$ $A = \frac{285}{360} \times \pi(8)^2 = 159 \text{ mm}^2$
Specific behaviours
✓ calculates internal sector angle ✓ calculates radius ✓ calculates area

(ii) Determine the perimeter of the sector. (2 marks)

Solution
Arc length: $L = \frac{285}{360} \times 2\pi(8) = 39.8$ Perimeter: $P = 39.8 + 2(8) = 55.8 \text{ mm}$
Specific behaviours
✓ calculates arc length ✓ calculates perimeter

Question 11 **(6 marks)**

The following table shows the exchange rates advertised by a bank for one Australian dollar:

Country (currency)	Poland (zloty, zl)	Hungary (forint, ft)	Kuwait (dinar)
Buy rate	3.034	251.2	0.2473
Sell rate	2.778	221.3	0.2249

When a person wants foreign currency, the bank will sell it to them using the sell rate. When a person wants to exchange foreign currency for Australian dollars, the bank will buy the currency from that person using the buy rate.

(a) An Australian collector of cars has found a model they are after in Poland for 142 000 zl and another in Hungary for 9 750 000 ft. Both sellers require payment in their local currency. Determine the price of each car in Australia dollars, rounding your answers to the nearest \$100. (3 marks)

Solution
$142\,000 \div 2.778 = 51\,115.91 \rightarrow$ Polish car will cost \$51 100. $9\,750\,000 \div 221.3 = 44\,057.84 \rightarrow$ Hungarian car will cost \$44 100.
Specific behaviours
✓ correctly divides both foreign amounts by either rate ✓ correctly divides both by sell rate ✓ both amounts correct and rounded

(b) A traveller changed 4400 Australian dollars into Kuwait dinars with this bank, spent 840 dinars whilst away in Kuwait and changed the remaining dinars back into Australian dollars on their return. Determine the number of Australian dollars they received. (3 marks)

Solution
Kuwait dinars: $4400 \times 0.2249 = 989.56$ Dinars returns with: $989.56 - 840 = 149.56$ Australian dollars: $149.56 \div 0.2473 = \$604.77$
Specific behaviours
✓ correctly converts to dinars ✓ calculates return amount ✓ correctly converts to dollars

See next page

Question 12 **(8 marks)**

The table below shows the GST inclusive prices for massage oil sold by a supermarket in three different sizes:

Size	Small	Medium	Large
Contents (mL)	450	750	1 200
Price ($)	5.85	10.20	14.52

(a) Determine the price per 100 mL for each size and hence explain which represents the best value, if price was the only consideration. (3 marks)

Solution
Small: $5.85 \div 4.5 = \$1.30$ Medium: $10.2 \div 7.5 = \$1.36$ Large: $14.52 \div 12 = \$1.21$ The large-size is best value as it costs the least per 100 mL.
Specific behaviours
✓ correct unit price for at least one size ✓ all correct unit prices ✓ states best value, with explanation

(b) The wholesale price paid by the supermarket for a box of 48 small-size oils is $\$224.64$, including GST. Determine the percentage profit it makes when selling this size. (3 marks)

Solution
Cost price per oil: $224.64 \div 48 = 4.68$ Profit per oil: $5.85 - 4.68 = 1.17$ % profit: $1.17 \div 4.68 = 25\%$
Specific behaviours
✓ indicates price per oil ✓ indicates profit per oil ✓ calculates percentage profit

(c) The rate of GST is 10%. Determine the amount of GST included in the price of a large-size massage oil. (2 marks)

Solution
Let x be pre-tax price, so that $1.1x = 14.52$ and hence $x = 14.52 \div 1.1 = 13.20$ Sales tax included is $14.52 - 13.20 = \$1.32$.
Specific behaviours
✓ indicates a suitable method ✓ calculates sales tax

See next page

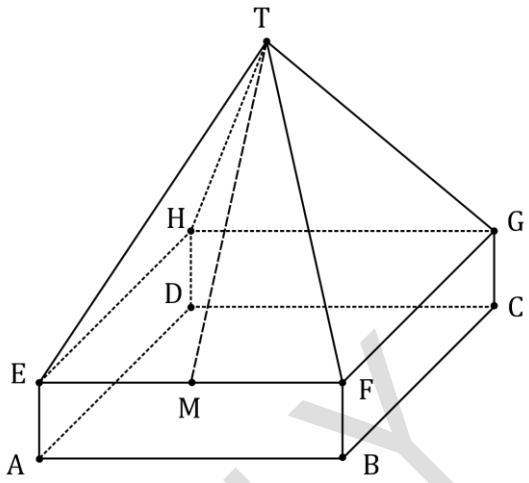
Question 13 **(8 marks)**

A solid object takes the form shown.

ABCDEFGH is a square prism with sides AB = BC = 48 cm and height $AE = 13$ m.

The apex T of pyramid EFGHT lies 45 cm directly above the centre of square EFGH.

The four sloping faces of the pyramid are congruent isosceles triangles, so that the length from T to M, the midpoint of edge EF is 51 cm.



(a) Determine the area of one of the triangular faces of the pyramid. (2 marks)

Solution
$A = \frac{1}{2} \times 48 \times 51$ $= 1224 \text{ cm}^2$
Specific behaviours
✓ indicates correct choice and use of formula ✓ calculates area

(b) Determine the surface area of the object. (3 marks)

Solution
One rectangular face: $48 \times 13 = 624$ Square base: $48 \times 48 = 2304$ Total surface area: $A = 2304 + 4 \times 624 + 4 \times 1224$ $= 9696 \text{ cm}^2$
Specific behaviours
✓ correctly calculates one relevant area ✓ clearly indicates correct method to obtain total area ✓ correct surface area

(c) Determine the volume of the object. (3 marks)

Solution
Pyramid: $V = \frac{1}{3}(2304)(45) = 34\,560$ Square prism: $V = 2304 \times 13 = 29\,952$ Hence total volume: $V = 34\,560 + 29\,952 = 64\,512 \text{ cm}^3$
Specific behaviours
✓ volume of pyramid ✓ volume of prism ✓ correct total volume

See next page

Question 14 **(7 marks)**

(a) An investor places $\$18\,750$ in a term deposit for 4 years. The term deposit pays interest of 5.5% compounded annually. Determine the total interest the investor will receive. (3 marks)

Solution

$$F = 18\,750\left(1 + \frac{5.5}{100}\right)^4$$
$$= 23\,227.96$$

$$I = 23\,227.96 - 18\,750$$
$$= \$\,4477.96$$

Specific behaviours

✓ expression for future value
✓ calculates future value
✓ calculates interest

(b) A short-term money lender offers simple interest loans at a daily interest rate of 0.0475%. A person borrows $\$800$ from this lender, paying back $\$350$ after 3 weeks and the remainder after a further 24 days. Determine the total interest on this loan. (4 marks)

Solution

(i) Interest for 3 weeks on $\$800$:

$$I = 800 \times \frac{0.0475}{100} \times 21 = \$7.98$$

(ii) Interest for 24 days on remainder:

$$P = 800 - 350 = 450$$

$$I = 450 \times \frac{0.0475}{100} \times 24 = 5.13$$

Total interest: $I = 7.98 + 5.13 = \$13.11$

Specific behaviours

✓ indicates partition into two amounts and times
✓ expression for one simple interest amount
✓ calculates one simple interest amount
✓ calculates second simple interest amount and total

See next page

Question 15 **(8 marks)**

An investor owns the following portfolio of stocks.

Stock	IFT	NAB	CSL	DXS
Number of shares owned	2500	800	200	1750
Share price ($)	6.98	23.25	287.79	9.59
Percentage dividend (%)	4.5	2.6	1.5	0
Earnings per share ($)	0.35	1.08	6.10	0.88

(a) Determine the value of the shares the investor owns in the highest priced stock. (2 marks)

Solution
CSL is highest price. Value: $287.79 \times 200 = \$57\,558$
Specific behaviours
✓ identifies stock ✓ calculates value of shares

(b) Calculate the total dividend paid on this share portfolio. (3 marks)

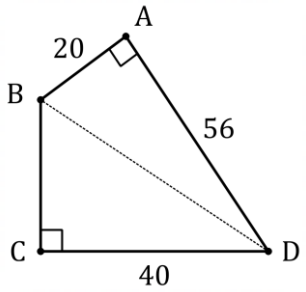
Solution
IFT: $2500 \times 6.98 \times 0.045 = 785.25$ NAB: $800 \times 23.25 \times 0.026 = 483.60$ CSL: $200 \times 287.79 \times 0.015 = 863.37$ DXS - no dividend Total: $785.25 + 483.60 + 863.37 = \$2132.22$
Specific behaviours
✓ one correct dividend ✓ second correct dividend ✓ all dividends and calculates total

(c) The investor plans to buy more shares in two of the four stocks. They will buy the stocks that have the lowest price-to-earnings ratio. State, with justification, which two stocks they will buy. (3 marks)

Solution
IFT: $6.98 \div 0.35 = 19.9$ NAB: $23.25 \div 1.08 = 21.5$ CSL: $287.79 \div 6.1 = 47.2$ DXS: $9.59 \div 0.88 = 10.9$ They will buy IFT and DXS.
Specific behaviours
✓ at least two correct P/E ratios ✓ all correct P/E ratios ✓ identifies two stocks as required

Question 16 **(8 marks)**

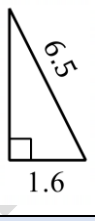
(a) The plan of a plot of land $ABCD$, not to scale, is sketched below with dimensions in metres. Determine the length of the diagonal BD, the length of side BC and hence the area of the plot. (4 marks)



Solution
$BD = \sqrt{56^2 + 20^2} = 59.5 \text{ m}$ $BC = \sqrt{59.5^2 - 40^2} = 44 \text{ m}$ Area: $\frac{1}{2}(56)(20) + \frac{1}{2}(40)(44) = 560 + 880 = 1440 \text{ m}^2$
Specific behaviours
✓ length BD ✓ length BC ✓ indicates sum of areas of two right triangles ✓ calculates area $ABCD$

(b) One end of a 6.5 m long ladder rests against a vertical wall and the other end stands on horizontal ground at a distance of 1.6 m from the foot of the wall.

(i) Sketch this situation and determine the height of the top of the ladder above the ground. (2 marks)

Solution
 $h = \sqrt{6.5^2 - 1.6^2} = 6.3 \text{ m}$
Specific behaviours
✓ labelled sketch ✓ correct height

(ii) If the foot of the ladder slides 110 cm across the ground and away from the wall, determine, to the nearest centimetre, how far the top of the ladder moves down the wall. (2 marks)

Solution
$h_2 = \sqrt{6.5^2 - 2.7^2} = 5.91 \text{ m}$ Ladder has slipped $630 - 591 = 39$ cm down wall.
Specific behaviours
✓ indicates new height above ground ✓ correct distance

See next page

Question 17 **(7 marks)**

A student who qualifies for a government allowance will receive a maximum payment of $462.50 per fortnight. If the student does any paid work and earns more than $437 per fortnight, the following reductions to the government payment apply:

Earnings	Amount payment reduces by
Between $437 and $524 per fortnight	50 cents for each dollar over $437
More than $524 per fortnight	$43.50 plus 60 cents for each dollar over $524

April qualifies for the allowance and has a casual job that pays her a fixed retainer of $23.50 per week plus $32.50 per hour worked.

(a) April will work 17 hours per week for the next fortnight. Determine

(i) her fortnightly earnings. (2 marks)

Solution
Weekly earnings: $17 \times 32.50 + 23.50 = \$576$ Fortnightly earnings: $2 \times 576 = \$1152$
Specific behaviours
✓ uses retainer and hourly rate ✓ correct earnings

(ii) her fortnightly government payment. (3 marks)

Solution
Excess: $1152 - 524 = 628$ Reduction: $43.50 + 0.6 \times 628 = 420.30$ Payment: $462.50 - 420.30 = \$42.20$
Specific behaviours
✓ calculates excess ✓ calculates reduction ✓ calculates payment

(b) Determine the maximum number of hours that April can work in a fortnight before her government payment will start to be reduced. (2 marks)

Solution
Maximum fortnightly payment less retainer: $437 - (2 \times 23.50) = 437 - 47 = \$390$ Paid work: $390 \div 32.50 = 12$ hours per fortnight.
Specific behaviours
✓ calculates maximum fortnightly earnings from wage ✓ correct hours per fortnight

See next page

Question 18 **(8 marks)**

The following spreadsheet was used to calculate the weekly pay of four office assistants. In this pay period, Friday was a public holiday, and the assistants were paid time-and-a-half that day.

	A	B	C	D	E	F	G	H
1	Employee	Base pay rate ($ per hour)	Daily hours worked					Weekly pay ($)
2			Mon	Tue	Wed	Thu	Fri	
3	Ian	24.75	8	0	8	7	4	717.75
4	Jem	26.50	5	?	0	6	0	450.50
5	Kai	33.50	0	8	5	8	5	954.75
6	Liam	30.25	6	4	6	0	0	?

(a) Show that the total pay for hours worked on Friday by the assistants came to $\$399.75$. (2 marks)

Solution
Ian: $24.75 \times 4 \times 1.5 = 148.50$ Kai: $33.50 \times 5 \times 1.5 = 251.25$ Total: $148.50 + 251.25 = \$399.75$
Specific behaviours
✓ shows wage for Ian ✓ shows wage for Kai and indicates sum

(b) Determine the missing value in cell H6 and the missing value in cell D4. (3 marks)

Solution
$H6 = (6 + 4 + 6) \times 30.25 = 484.00$ $(5 + x + 6) \times 26.50 = 450.50 \rightarrow x = D4 = 6$
Specific behaviours
✓ value of H6 ✓ indicates equation using missing value D4 ✓ value of D4

(c) By referring to the value in a spreadsheet cell using its column letter followed by its row number (e.g., B4 refers to the value 26.50 in that cell) write a formula to

(i) recalculate the value in cell H3 when any cell value in the spreadsheet is changed (1 mark)

Solution
$H3 = B3 \times (C3 + D3 + E3 + F3 + G3 \times 1.5)$
Specific behaviours
✓ any correct formula ($H3 =$ not required)

(ii) place in cell H7 that will calculate the total weekly pay for the assistants, and state what this total is. (2 marks)

Solution
$H7 = H3 + H4 + H5 + H6$ or $H7 = \text{SUM}(H3{:}H6)$ Total: $717.75 + 450.50 + 954.75 + 484.00 = \$2607.00$
Specific behaviours
✓ any correct formula ($H7 =$ not required) ✓ correct total weekly pay

See next page

Question 19 **(8 marks)**

The following diagram, drawn to scale, shows the plan for a new rectangular lawn on a plot of level ground. The lawn is 6 metres wide.

(a) Determine, to the nearest metre, the length of the lawn. (2 marks)

Solution
On diagram, width is 30 mm and length is 90 mm. $L = \frac{90}{30} \times 6 = 18$ m
Specific behaviours
✓ indicates use of appropriate method ✓ correct length

(b) Determine the total cost of turf to cover the lawn, given that suitable turf costs $\$16.25$ per square metre. (2 marks)

Solution
$A = 6 \times 18 = 108\ \text{m}^2$ $\text{Cost} = 108 \times 16.25 = \$1755$
Specific behaviours
✓ indicates area of lawn ✓ correct cost

(c) Express the scale factor of the drawing in the form $1:n$. (2 marks)

Solution
$30\ \text{mm} : 6\ \text{m} \rightarrow 30 : 6000 \rightarrow 1:200$
Specific behaviours
✓ correctly eliminates units ✓ correct scale factor

(d) The turf will be laid on top of a layer of special soil that has a uniform depth of 75 mm. Determine the volume of soil that is required. (2 marks)

Solution
$V = 108 \times 0.075$ $V = 8.1\ \text{m}^3$
Specific behaviours
✓ converts soil thickness to metres ✓ correct volume of soil with units

See next page

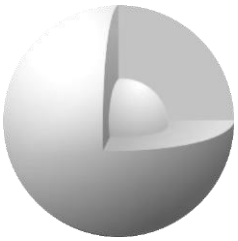
Question 20 **(8 marks)**

A company makes spherical floats with an external radius of 30 cm for use in boat harbours.

(a) The external surface of a float is coated with a special epoxy paint that costs \$95 per square metre. Determine the cost of this paint for one float. (3 marks)

Solution
$r = 0.30$ m Surface area: $A = 4\pi(0.30)^2$ $= 1.131$ m^2 Cost: $C = 1.131 \times 95 = \$107.44$
Specific behaviours
✓ uses radius in metres ✓ calculates surface area ✓ calculates cost, rounded to nearest cent

(b) Each float is made from an inner solid steel sphere of radius 12 cm surrounded by a thick layer of polystyrene, as shown in the cutaway diagram.



Given that the weights of one cubic centimetre of steel and polystyrene are 6.2 and 0.025 grams respectively, determine the total weight of materials in one float.

(5 marks)

Solution
Whole volume: $V = \frac{4}{3}\pi(30)^3 = 113\,097$ cm^3 Steel volume: $V_S = \frac{4}{3}\pi(12)^3 = 7238$ cm^3 Polystyrene volume: $V_P = 113\,097 - 7238 = 105\,859$ cm^3 Steel weight: $W_S = 7238 \times 6.2 = 44\,877$ g Polystyrene weight: $W_P = 105\,859 \times 0.025 = 2646$ g Total weight: $W = 44\,877 + 2646 = 47\,523$ g
Specific behaviours
✓ calculates volume of whole sphere ✓ calculates volume of inner sphere ✓ calculates volume of polystyrene ✓ calculates weight of at least one material ✓ calculates total weight and states units

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Question 21 **(8 marks)**

When a sum of $\$1400$ is invested at a rate of R percent per annum, compounded every two months, the total interest I accumulated after T years is given by the formula

$$I = 1400\left(\left(1+\frac{R}{600}\right)^{6T}-1\right)$$

This formula has been used to create the spreadsheet below.

	A	B	C	D	E	F	G
1	Interest, I		Rate, R				
2			1	2	3	4	5
3	Time, T	1	14.06	28.23	42.53	56.94	71.47
4		2	28.26	57.04	86.35	116.20	146.60
5		3	42.60	86.42	131.50	177.87	225.56
6		4	57.09	116.4	178.02	242.04	?

(a) When the rate is 3% per annum, determine

(i) the total interest accumulated after 3 years. (1 mark)

Solution
$\$131.50$
Specific behaviours
✓ correct value

(ii) the interest accumulated during the fourth year. (2 marks)

Solution
$178.02 - 131.50 = \$46.52$
Specific behaviours
✓ uses difference of years
✓ correct value

(b) Determine, to the nearest cent, the total interest after 4 years at a rate of 5% per annum. (2 marks)

Solution
$I = 1400\left(\left(1+\frac{5}{600}\right)^{6\times4}-1\right) = \$308.55$
Specific behaviours
✓ substitutes correctly
✓ correct value, to nearest cent

(c) If the spreadsheet was extended, determine the value that would appear in cell K8. (3 marks)

Solution
In cell K8, $R = 9$ and $T = 6$
$I = 1400\left(\left(1+\frac{9}{600}\right)^{6\times6}-1\right) = \$992.80$
Specific behaviours
✓ indicates correct values for R and T
✓ substitutes correctly
✓ correct value

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____